Cosmological perturbations and noncommutative tachyon inflation

Dao-jun Liu and Xin-zhou Li*
Shanghai United Center for Astrophysics(SUCA),
Shanghai Normal University, 100 Guilin Road, Shanghai 200234,China
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ABSTRACT: The motivation for studying the rolling tachyon and non-commutative inflation comes from string theory. In the tachyon inflation scenario, metric perturbations are created by tachyon field fluctuations during inflation. We drive the exact mode equation for scalar perturbation of the metric and investigate the cosmological perturbations in the commutative and non-commutative inflationary spacetime driven by the tachyon field which have a Born-Infeld Lagrangian.

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I. INTRODUCTION

The cosmological parameters and the properties of inflationary models are tightly constraint by the recent result from Wilkinson Microwave Anisotropy Probe (WMAP)[1] and other earlier observations. The standard inflationary ΛCDM model provides a good fit to the observed cosmic microwave background (CMB) anisotropies. The first-year results of WMAP also bring us something intriguing, some analyses [2, 3, 4, 5] show that the new data of CMB suggesting an anomalously low quadrupole and octupole and a larger running of the spectral index of the power spectrum than that predicted by standard single scalar field inflation models satisfying the slow roll conditions.

One typically considers an inflationary phase driven by the potential of the inflation , whose dynamics is determined by a canonical scalar action. Recently, pioneered by Sen [6], the study of non-BPS objects such as non-BPS branes, brane-antibrane configurations or space-like branes has attracted physical interests in string theory. Sen showed that classical decay of unstable D-brane in string theories produces pressureless gas with non-zero energy density. Gibbons took into account the gravitational coupling by adding an Einstein-Hilbert term to the effective action of the tachyon on a brane, and initiated a study of "tachyon cosmology" [7]. This provides a rich gamut of possibilities in the context of cosmology, including slow-roll inflation [8]. As an inflationary mechanism, tachyon condensation has been criticized by some authors [9]. Their reason is that for string theory motivated values of the parameters in potential $V(T)$, there is an incompatibility between the slow-roll condition and COBE normalization of fluctuations. However, the potential can be found for which this issue may be circumvented [10]. Thus, one can take a phenomenological approach and study the inflationary predictions by the tachyon field. Tachyon inflation leads to a deviation in one of the second order consistency relations, and its predictions are typically characteristic of small field or chaotic inflation [11]. All the typical tachyon models predict a negative and very small running of the scalar spectral index, and they consistently lie within the 1σ contour of the data set. However, the regime of blue scalar spectral index and large gravitational waves is not explored by these model.

On the other hand, it is well known that during the period of inflation, the classical gravitational theory, general relativity, might break down due to the very high energies at that time and the correction from string theory may take effect. In the nonperturbative string/M theory, any physical process at the very short distance takes an uncertainty relation, called stringy spacetime uncertainty relation (SSUR),

$$\Delta t_p \Delta x_p \geq l_s^2, \tag{1}$$

where t_p and x_p are the physical time and space, l_s is the string length scale. It is suggested that the SSUR is a universal property for strings as well as D-branes [12]. Unfortunately, we now have no ideas to derive cosmology directly from string/M theory. Brandenberger and Ho [13] have proposed a variation of spacetime non-commutative field theory to realize the stringy spacetime uncertainty relation without breaking any of the global symmetries of the homogeneous isotropic universe. If inflation is affected by physics at a scale close to string scale, one expects that spacetime uncertainty must leave vestiges in the CMB power spectrum [14, 15, 16, 17]. It is found that the modification from the non-commutative spacetime or SSUR delayed the cosmological perturbation mode crosses the Hubble horizon for a smaller Hubble constant and thus suppress the fluctuation which implies that the running of the spectral index is larger than the one in the commutative case.

The motivation for studying the tachyon inflation comes from type II string theory. It is not unique, but has its counterpart. The non-commutative inflation which takes into account some effects of the spacetime uncertainty principle motivated by ideas from string theory. Therefore, in this paper, we investigate cosmological perturbations of the metric during the tachyon inflation in non-commutative spacetime. Using the "Mukhanov variable" z, after a prolix but straightforward calculation, we show the exact mode equation for the scalar perturbation

*Electronic address: kychz@shnu.edu.cn

of metric. In the tachyon inflation scenario, we conform that the non-commutative spacetime effects always suppress the power spacetime of both the scalar and tensor perturbations, and may provide a large enough running of the spectral index to fit the WMAP data.

II. HAMILTON-JACOBI EQUATION OF TACHYON INFLATION

The flat FRW line element is given by:

$$\begin{aligned} ds^2 = &\quad dt^2 - a^2(t)(dx^2+dy^2+dz^2) \\ = &\quad a^2(\tau)[d\tau^2 - (dx^2+dy^2+dz^2)] \end{aligned} \tag{2}$$

where τ is the conformal time, with $dt = ad\tau$. The Lagrangian density of a rolling tachyon is

$$L = \sqrt{-g}\left(\frac{R}{2\kappa} - V(T)\sqrt{1-g^{\mu\nu}\partial_\mu T\partial_\nu T}\right) \tag{3}$$

where $\kappa = 8\pi G = M_p^{-2}$. For a spatially homogenous tachyon field T, we have the equation of motion

$$\ddot{T} + 3H\dot{T}\left(1-\dot{T}^2\right) + \frac{V'}{V}\left(1-\dot{T}^2\right) = 0 \tag{4}$$

which is equivalent to the entropy conservation equation. Here, the Hubble parameter H is defined as $H \equiv (\frac{\dot{a}}{a})$, and $V' = dV/dT$. If the stress-energy of the universe is dominated by the tachyon field T, the Einstein field equations for the evolution of the background metric, $G_{\mu\nu} = \kappa T_{\mu\nu}$, can be written as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\frac{V(T)}{\sqrt{1-\dot{T}^2}} \tag{5}$$

and

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = \frac{\kappa}{3}\frac{V(T)}{\sqrt{1-\dot{T}^2}}\left(1-\frac{3}{2}\dot{T}^2\right) \tag{6}$$

Eqs.(4)-(6) form a coupled set of evolution equations of the universe. The fundamental quantities to be calculated are $T(t)$ and $a(t)$, and the potential $V(T)$ is given when the model is specified. The period of accelerated expansion corresponds to $\dot{T}^2 < \frac{2}{3}$ and decelerate otherwise. In the limit case $\dot{T} = 0$, there is no difference in meaning of the expansion of universe between tachyon inflation and ordinary inflation driven by inflaton. However, the case of $\dot{T} \neq 0$ forms a sharp contrast. Although the formulas of tachyon inflation are correspond to those of the inflation driven by ordinary scalar field, there is obvious difference between them which can not be neglected. From Eqs.(4)-(6), we have two first-order equations

$$\dot{T} = -\frac{2}{3}\frac{H'(T)}{H^2(T)} \tag{7}$$

$$[H'(T)]^2 - \frac{9}{4}H^4(T) = -\frac{\kappa^2}{4}V^2(T) \tag{8}$$

These equations are wholly equivalent to the second-order equation of motion (4).

Analogous to the inflation driven by ordinary scalar field, for example in Ref.[18], we need define the "slow-roll" parameters. In general, there are two ways to define them. One is that we can take the definitions are independent of field driving inflation [11], $\epsilon \equiv H_*/H$ and $\epsilon_{i+1} = d\ln|\epsilon_i|/dN$ $(i \geq 0)$, where H_* is the Hubble parameter at some chosen time. They are an advantageous choice to use in order to compare ordinary and tachyon inflation, though the observables (such as spectral indices) will no longer be related to the parameters in the same way. And the other is called tachyonic slow-roll parameters as follows

$$\epsilon(T) \equiv \frac{2}{3}\left(\frac{H'(T)}{H^2(T)}\right)^2, \tag{9}$$

$$\eta(T) \equiv \frac{2}{3}\left(\frac{H''(T)}{H^3(T)}\right), \tag{10}$$

$$\xi(T) \equiv \frac{2}{3}\left(\frac{H'(T)H'''(T)}{H^6(T)}\right)^{1/2}. \tag{11}$$

Obviously, the definitions of the parameters Eqs.(9)-(11) are quite different from those defined in ordinary inflation. This is very natural for the Born-Infeld action is sharply different from that of the ordinary scalar field. In next section these parameters will be conveniently applied to exact mode equation of tachyon inflation. In term of $\epsilon(T)$ parameter, Eq.(8) can be reexpressed as

$$H^4(T)[1-\frac{1}{3}\epsilon(T)] = \frac{\kappa^2}{9}V^2(T) \tag{12}$$

which is referred to as the Hamilton-Jacobi equation of tachyon inflation. Using Eq.(7), we have

$$\epsilon(T) = \frac{3}{2}\dot{T}^2. \tag{13}$$

Note that the Hamilton-Jacobi equation has the same form as that of the ordinary inflaton field only up to first order term in $\epsilon(T)$. This can be found by comparing the Hamilton-Jacobi equation for an ordinary scalar field [19] with the one for the tachyon, Eq.(12),

$$H^2\left[1-\frac{1}{3}\epsilon(T)\right] = \frac{\kappa}{3}\frac{(1-\frac{1}{2}\dot{T}^2)V(T)}{\sqrt{1-\dot{T}^2}} = \frac{8\pi G}{3}V(T) + O(\epsilon^2) \tag{14}$$

The number of e-folds of the inflation produced when the tachyon field rolls from a particular value T to the end point T_e is

$$N(T,T_e) \equiv \int_t^{t_e} H(t)dt = \int_T^{T_e}\frac{H}{\dot{T}}dT \tag{15}$$

Therefore, we have

$$a(T) = a_e\exp[-N(T)] \tag{16}$$

where a_e is the value of the scale factor at the end of inflation. Since after tachyon inflation the dynamics of the reheating is still unclear, in the following we shall typically assume a conservative value of e-folds $40 \leq N \leq 70$ [11].

Given a non-commutative spacetime that obeys the stringy spacetime uncertainty relation, the cosmological background will still be described by the Einstein equations since the background fields only depend on one spacetime variable [13]. Thus, the formula for the tachyon field that drives the non-commutative spacetime inflating have the same form as in the ordinary commutative spacetime. But the equations for the linear fluctuations should be modified. Brandenberger and Ho [13] argued that the modifications take the form of an interaction of the fluctuating field with the background which is nonlocal in time.

III. THE COSMOLOGICAL PERTURBATIONS IN COMMUTATIVE SPACETIME

During inflation, quantum fluctuations are stretched on scale larger than the horizon. There they are frozen until they reenter the horizon after inflation. Regardless of the field which drives inflation, a quasi scale invariant spectrum are generated for large scale perturbations. The most important observational test of inflation is observation of the Cosmic Microwave Background (CMB) radiation. Temperature fluctuations in the CMB can be related to perturbations in the metric at the surface of last scattering. The metric perturbations are created by tachyon fluctuations during inflation. In the inflation scenario, quantum fluctuations on small scales are rapidly red-shifted to scales much larger than the horizon size. The metric perturbations can be decomposed according to their spin with respect to a local rotation of the spatial coordinates on hypersurfaces of constant time. This leads to two types: scalar, or curvature perturbations, which couple to the tachyon and form the "seeds" for structure formation, and tensor, or gravitational wave perturbations, which do not couple to tachyon. Both scalar and tensor perturbations contribute to CMB anisotropy.

Considering small fluctuations of the tachyon field, that is

$$T(t, \mathbf{x}) = T_0(t) + \delta T(t, \mathbf{x}) \tag{17}$$

and one can take the metric of the "perturbed universe" in the longitudinal gauge as [20]

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)a^2(t)\delta_{ij}dx^i dx^j \tag{18}$$

where Φ is the newtonian gravitational potential. The linearized Einstein equations can be written as

$$\dot{\chi} = \frac{a}{H^2}\frac{V(T)\dot{T}^2}{\sqrt{1-\dot{T}^2}}\zeta \tag{19}$$

$$\dot{\zeta} = \frac{H^2}{a^3}\frac{(1-\dot{T}^2)^{3/2}}{V(T)\dot{T}^2}\nabla^2\chi \tag{20}$$

where the new variables χ and ζ are respectively defined as

$$\chi \equiv \frac{2a}{\kappa^2 H}\Phi, \qquad \zeta \equiv \Phi + H\frac{\delta T}{\dot{T}}. \tag{21}$$

The intrinsic curvature perturbation of the comoving hypersurfaces ζ is gauge invariant. It is not difficult to show that one can relate the fluctuation of the gravitational potential Φ to the fluctuation of the tachyon field δT on superhorizon scales. The canonical quantization variable u is defined as $u \equiv z\zeta$, where

$$z = \frac{a}{H}\sqrt{\frac{V(T)\dot{T}^2}{(1-\dot{T}^2)^{3/2}}}, \tag{22}$$

is the so-called "Mukhanov variable" [21]. From (19) and (20), we have

$$\left[az^2\left(\frac{u}{z}\right)^{\cdot}\right]^{\cdot} = \frac{z}{a}(1-\dot{T}^2)\nabla^2 u. \tag{23}$$

Using the conformal time τ instead of physical time t, after a prolix but straightforward calculation, Eq.(23) can be reduced to

$$\frac{d^2u}{d\tau^2} - (1 - \frac{2}{3}\epsilon)\nabla^2 u - \frac{1}{z}\frac{d^2z}{d\tau^2}u = 0, \tag{24}$$

where

$$\begin{aligned}\frac{1}{z}\frac{d^2z}{d\tau^2} = 2a^2H^2\Big[1 &+ \frac{1}{(1-\frac{2}{3}\epsilon)^2}\Big(\frac{5}{2}\epsilon - \frac{3}{2}\eta \\ &+ \frac{11}{2}\epsilon^2 - 4\epsilon\eta + \frac{1}{2}\eta^2 + \frac{1}{2}\xi^2 \\ &+ \frac{4}{3}\epsilon^2\eta + \frac{2}{3}\epsilon\eta^2 - \frac{1}{3}\epsilon\xi^2\Big)\Big]. \end{aligned} \tag{25}$$

As usual, the zeroth order term $2a^2H^2$ ensures that the spectrum is scale invariant. Expanding the quantity u in Fourier modes

$$u(\tau, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}u_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{26}$$

the mode function u_k satisfies the following equation

$$\frac{d^2u_k}{d\tau^2} + \left[(1-\frac{2}{3}\epsilon)k^2 - \frac{1}{z}\frac{d^2z}{d\tau^2}\right]u_k = 0. \tag{27}$$

Therefore, the spectrum of curvature perturbation $P_R(k)$ as function of wavenumber k could be expressed as

$$P_R^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}}\left|\frac{u_k}{z}\right|. \tag{28}$$

Clearly, the above expression Eq.(25) is different from that of the ordinary scalar field because the coupling of curvature perturbations to the stress-energy of tachyon field is in a very different manner.

The Born-Infeld action is quite different from that of the ordinary scalar field. Therefore, although the expressions of tachyon inflation correspond to those of the inflation driven by the ordinary scalar field, there is obvious difference between them, which can not be neglected. We find that the usual relation between the scalar and tensor spectral index is modified. Therefore, at least in principle, tachyon inflation is distinguishable from standard inflation. Here, it is worth noting that some authors [11] have obtained the mode equation of tachyon inflation where the slow-roll approximation is appealed from the beginning of the reduction, but the expressions given above are all exact without slow roll approximation. Especially, at lowest order of slow roll formalism the predictions of ordinary and tachyon inflation are shown to be the same. Higher order deviation are present in Eq.(25).

IV. PERTURBATION SPECTRUM IN NON-COMMUTATIVE TACHYON INFLATION

The action which reproduces the equation of motion (24) can be written as

$$S=\frac{1}{2}\int d\tau d^3xz^2[(\partial_\tau\zeta)^\dagger(\partial_\tau\zeta)-c_s^2(\nabla\zeta)^\dagger(\nabla\zeta)], \quad (29)$$

where intrinsic curvature perturbation ζ and mode function u have the relationship $z\zeta=u$, and c_s denotes sound velocity which determined by background tachyon field. In the momentum space, Eq. (29) can be rewritten as

$$S=\frac{1}{2}V_T\int d\tau d^3kz^2[(\partial_\tau\zeta_{-k})(\partial_\tau\zeta_k)-c_s^2k^2\zeta_{-k}\zeta_k], \quad (30)$$

where V_T is the total spatial coordinate volume and k is the comoving wave number. Following the similar process proposed in Ref.[13], the SSUR is compatible with an unchanged homogeneous background, but it leads to changes in the action for the metric fluctuation. We get a model with non-commutative modifications

$$S=\frac{1}{2}V_T\int d\tilde{\tau}d^3kz_k^2[\zeta'_{-k}\zeta'_k-c_s^2k^2\zeta_{-k}\zeta_k], \quad (31)$$

where z_k is defined as

$$z_k=(\beta_k^+\beta_k^-)^{1/4}z, \quad (32)$$

in which z is the "Mukhanov variable" defined in Eq.(22), and

$$\beta_k^\pm=\frac{1}{2}[a^{\pm2}(\hat{\tau}+kl_s^2)+a^{\pm2}(\hat{\tau}-kl_s^2)]. \quad (33)$$

Here, the new time variable $\hat{\tau}$ is defined as $d\hat{\tau}=a^2d\tau$. The primes appeared in the Eq.(31) denote the derivative with respect to the new time variable $\tilde{\tau}$, and $\tilde{\tau}$ is related to the conformal time τ via

$$d\tilde{\tau}=a^2\left(\frac{\beta_k^-}{\beta_k^+}\right)^{1/2}d\tau. \quad (34)$$

Apparently, if the string length scale l_s goes to zero, the action (31) will reduce to the action (30) for the fluctuations in the classical spacetime, which leads to the equation of motion of perturbations (27). From the action (31), the equation of motion of the scalar perturbations can be written as

$$u_k''+\left(c_s^2k^2-\frac{z_k''}{z_k}\right)u_k=0, \quad (35)$$

where the mode function is defined by $u_k=z_k\zeta_k$ and the sound velocity c_s satisfies

$$c_s^2=1-\frac{2}{3}\epsilon. \quad (36)$$

Let

$$\lambda=\frac{H^2k^2}{a^2M_s^4}, \quad (37)$$

where k is the comoving wave number of a perturbation mode, and $M_s=l_s^{-1}$ is the string mass scale. λ is a small dimensionless quantity, because we assume the string mass scale M_s is very large. Using the slow-roll parameters and $\frac{1}{z}\frac{d^2z}{d\tau^2}$ in Eq.(25), we get

$$\begin{aligned}\frac{z_k''}{z_k} &= \frac{1}{z}\frac{d^2z}{d\tau^2}[1-2(1+\epsilon)\lambda]\\ &+ 2a^2H^2\lambda\left[3\epsilon\eta-2\epsilon^2+5\epsilon+1+\frac{3\epsilon(2\epsilon-\eta)(\eta-\epsilon+1)}{3-2\epsilon}\right],\end{aligned} \quad (38)$$

up to the first order of λ. Clearly, when $l_s\to0$ or $M_s\to\infty$, the quantity z_k''/z_k and $\tilde{\tau}$ will be reduced to $\frac{1}{z}\frac{d^2z}{d\tau^2}$ and τ respectively, and then the motion equation (35) of the mode μ_k in noncommutative spacetime will recover the one in ordinary commutative spacetime (27).

In the slow-roll approximation, the conformal time τ can be expressed approximately by

$$\tau\simeq-\frac{1+\epsilon}{aH}. \quad (39)$$

From Eq.(34), we have

$$\tau\simeq(1-\lambda)\tilde{\tau}, \quad (40)$$

and then,

$$\tilde{\tau}\simeq-\frac{1}{aH}(1+\epsilon+\lambda). \quad (41)$$

On the other hand, up to the first order of slow-roll parameters, Eq. (38) can be approximated by

$$\frac{z_k''}{z_k}=2a^2H^2\left(1+\frac{5}{2}\epsilon-\frac{3}{2}\eta-\lambda\right). \quad (42)$$

Using Eqs.(41) and (42), we rewrite the equation of motion for scalar fluctuation mode (35) as

$$u_k''+\left[c_s^2k^2-\frac{\mu^2-\frac{1}{4}}{\tilde{\tau}^2}\right]u_k=0, \quad (43)$$

where the parameter

$$\mu \simeq \frac{3}{2} + 3\epsilon - \eta + \frac{2}{3}\lambda. \quad (44)$$

These modes are normalized so that they satisfy the Wronskian condition

$$u_k^* \frac{du_k}{d\tilde{\tau}} - u_k \frac{du_k^*}{d\tilde{\tau}} = -i. \quad (45)$$

On the subhorizon scale $c_s^2 k^2 \gg z_k''/z_k$, the equation (43) has a plane wave solution

$$u_k = \frac{1}{\sqrt{2c_s k}} e^{-ic_s k\tilde{\tau}}. \quad (46)$$

which indicates that perturbation with wavelength within the horizon oscillate like in flat spacetime. This does not come as a surprise, since in the UR regime, one expects that approximating the spacetime as flat is a good approximation.

Taking the solution (46) as the initial condition, we can obtain the solution of (43) on the superhorizon, $c_s^2 k^2 \ll z_k''/z_k$,

$$\begin{aligned} u_k &\simeq \frac{1}{\sqrt{2c_s k}} (-c_s k\tilde{\tau})^{\frac{1}{2}-\mu} \\ &\simeq \frac{1}{\sqrt{2c_s k}} \left[\frac{c_s k(1+\epsilon+\lambda)}{aH}\right]^{\frac{1}{2}-\mu}. \end{aligned} \quad (47)$$

Thus, we can express the power spectrum on superhorizon scales of the comoving curvature as

$$\begin{aligned} P_R(k) &= \frac{k^3}{2\pi^2} \left|\frac{u_k}{z_k(\tilde{\tau})}\right|^2 \\ &\simeq \frac{1}{2\epsilon} \frac{1}{M_{pl}^2} \left(\frac{H}{2\pi}\right)^2 \left(\frac{c_s k}{aH}\right)^{3-2\mu} (1+\lambda)^{-1-2\mu} \end{aligned} \quad (48)$$

When the perturbation mode k crosses the Hubble radius,

$$c_s^2 k^2 = \frac{z_k''}{z_k} = 2a^2H^2 \left(1 + \frac{5}{2}\epsilon - \frac{3}{2}\eta - \lambda\right). \quad (49)$$

At the same time, the power spectrum is reduced to be

$$\begin{aligned} P_R(k) &\simeq \frac{1}{2\epsilon} \frac{1}{M_{pl}^2} \left(\frac{H}{2\pi}\right)^2 (2 + 5\epsilon - 3\eta - 2\lambda)^{-3\epsilon+\eta-\frac{2}{3}\lambda} \\ &\times (1+\lambda)^{-4-6\epsilon+2\eta-\frac{4}{3}\lambda}. \end{aligned} \quad (50)$$

Up to the first order of slow-roll parameters and λ, we obtain

$$\frac{d\ln k}{dt} \simeq (1 - \epsilon + 4\epsilon\lambda)H, \quad (51)$$

and

$$\frac{d\lambda}{d\ln k} \simeq -4\epsilon\lambda. \quad (52)$$

Therefore, the spectra index of the scalar metric perturbation and its running can be expressed respectively as follows:

$$n_s - 1 \equiv \frac{d\ln P_R}{d\ln k} \simeq -6\epsilon + 2\eta + \frac{8(6+\ln 2)}{3}\epsilon\lambda, \quad (53)$$

$$\frac{dn_s}{d\ln k} = 14\epsilon\eta - 24\epsilon^2 - 2\xi^2 + \frac{16}{3}(6+\ln 2)\epsilon\eta\lambda. \quad (54)$$

Obviously, when the parameter $\lambda \to 0$, the contribution from the non-commutativity of spacetime to the spectral index and its running will also vanish. Similar to the case in the ordinary noncommutative inflation [17], the effects of the non-commutativity of spacetime suppress the power spectrum of the primordial perturbations which lead to a more blue spectrum with a correction $\frac{8(6+\ln 2)}{3}\epsilon\lambda$ to the spectrum index.

V. DISCUSSION

Just as the ordinary inflation scenario, besides the scalar perturbations that couple to the matter distribution in the universe and form the "seeds" of the large scale structure, tachyon inflation both in commutative and non-commutative spacetime also predicts a tensor perturbation, or called gravitational perturbation, of the metric. Because the tensor perturbations during the period of inflation depend only on the energy scale of the inflation, we can consider that they only describe the propagation of gravitational waves and do not couple to the matter term. Therefore, the expressions that describe the same as that in ordinary scalar field inflation.

Steer and Vernizzi [11] have investigated typical inflationary tachyon potentials, such as the inverse cosh potential, the exponential potential and the inverse power law potential. They also discussed their observational consequences and compared them with WMAP data. The regime of blue scalar spectral index and large gravitational waves is not explored by these potentials. However, the effects of the non-commutativity of spacetime suppress the power spectrum of the primordial perturbations which leads to a more blue spectrum for the tachyon inflation scenario.

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